Georgia Standards of Excellence Curriculum Frameworks

Mathematics

GSE Grade 7

Unit 5: Inferences

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Unit 5
Inferences

OVERVIEW

The units in this instructional framework emphasize key standards that assist students in developing a deeper understanding of numbers. They have learned to recognize and express different representations of rational numbers. Now they will learn how to apply the statistical aspect of mathematics to the measures of center, variability and population. The Big Ideas that are expressed in this unit are integrated with such routine topics as estimation, mental and basic computation. All of these concepts need to be reviewed throughout the year.
The Evidence of Learning will tell you what your students will learn in this unit. Take what you need from the tasks and modify as required. These tasks are suggestions, something that you can use as a resource for your classroom.

STANDARDS ADDRESSED IN THIS UNIT

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of information by connecting visual, tabular, and symbolic representations of sample populations in real-life contexts.

2. Reason abstractly and quantitatively. Students' reason about the values in data representations based upon their relationship to the real number line.

3. Construct viable arguments and critique the reasoning of others. Students use data to make inferences from sample sets. They construct viable arguments by referring to representations as evidence of their inferences and question each other regarding these inferences.

4. Model with mathematics. Students generate representative samples in real-world contexts and represent these visually, in tables, and symbolically to gain information from sample sets.

5. Use appropriate tools strategically. Students choose appropriate mathematical and visual representations, including technology-based tools, to represent the data distributions.

6. Attend to precision. Students use precision to collect accurate measurement information from sample populations and precise language when generating and interpreting data.

7. Look for and make use of structure. Students interpret data representations in tables, histograms, box plots and scatter plots by examining the features of those representations.

8. Look for and express regularity in repeated reasoning. Students look to make generalized comparisons between situations that involve bias using specific criteria.

STANDARDS FOR MATHEMATICAL CONTENT

Use random sampling to draw inferences about a population.

MGSE7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

MGSE7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Draw informal comparative inferences about two populations.

MGSE7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the medians by expressing it as a multiple of the interquartile range.

MGSE7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

BIG IDEAS

- Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population.
- Understand that random sampling tends to produce representative samples and support valid inferences.
- Use data from a random sample to draw inferences about a population with an unknown characteristic of interest.
- Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.
- Informally assess the degree of visual overlap of two numerical data distributions with similar variability, measuring the difference between the centers by expressing it as a multiple of a measure of variability.
- Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

ESSENTIAL QUESTIONS

- What are different sampling techniques used in real life?
- How do I determine an appropriate sample size?

- How can random samples be used to make predictions about populations?
- How are proportions used to estimate information about populations?
- How does the data describe its center, spread and representation of the population?
- What is the difference between the measure of center and measure of variation?
- How do I display data on a number line?
- What are ways to summarize numerical data sets?

CONCEPTS AND SKILLS TO MAINTAIN

In order for students to be successful, the following skills and concepts need to be maintained

- Analyzing patterns and seeing relationships
- Represent and interpret data, using addition and subtraction, multiplication and division
- Data can be represented graphically in a variety of ways. The type of graph is selected to best represent a particular data set.
- Measures of center (mean, median, mode) and measures of variation (range, quartiles, interquartile range) can be used to analyze data.
- Larger samples are more likely to be representative of a population.

FLUENCY

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.
Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures and/ or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.** The definitions below are from the CCSS glossary http://www.corestandards.org/Math/Content/mathematics-glossary/glossary, when applicable.

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 7 mathematics.

- **Box and Whisker Plot**
- **Frequency**
- **Grouped Frequency Table**
- **Histogram**
- **Inter-Quartile Range (IQR)**
- **Maximum value**
- **Mean**
- **Measures of Center**
- **Measures of Spread**
- **Median**
- **Minimum value**
- **Mode**
- **Mutually Exclusive**

- **Outlier**
- **Range**
- **Sample**
- **Simple Random Sampling**
- **Stem and Leaf Plot**

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

A Spotlight Task has been added to each MGSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Mathematics Georgia Standards of Excellence, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

TASKS

Task Name	Task Type *Grouping Strategy*	Content Addressed	Standards Addressed
Is It Valid?	Learning Task *Individual/Partner*	Scenarios to help determine if a sample is valid and representative of the population	MGSE7.SP.1
Snapshot	Learning Task *Partner/Small Group*	Determine how many tiles of each color are in a population.	MGSE7.SP.1 MGSE7.SP.2
Candy Populations	Performance Task *Small Group*	Analyze populations and samples in order to make predictions about a larger population	MGSE7.SP.1 MGSE7.SP.2
Predicting Population	Performance Task *Individual / Partner*	Using population samples to make predictions about a given set of data	MGSE7.SP.1 MGSE7.SP.2
Counting Trees	Problem Solving Task *Partner/SmallGroup*	Use samples to make estimations about a larger population	MGSE7.SP.1 MGSE7.SP.2 MGSE7.SP.3 MGSE7.SP.4
Human Box Plot	Scaffolding Task *Whole group*	Construct a box plot using given data.	MGSE7.SP.3
Shakespeare vs. Harry Potter	Learning Task *Partner or with teacher as a whole group task*	Compare the word length in passages of Shakespeare to the length of the words in Harry Potter. Focus on mean, 5 number summary, box plots, comparing evidence and finding comparing data sets.	MGSE7.SP.2
Box and Whisker Plots (FAL)	Formative Assessment Lesson *Partner/SmallGroup*	Interpretation of box and whisker plots to evaluate sets of data; assimilation of multiple sets of data related	MGSE7.SP.3 MGSE7.SP.4

		to a single entity; making value judgments, particularly when information found in the data seems to be contradictory; comparison of entities and making decisions about "good," "better," and "best" based upon distribution of summary data (medians, quartiles, interquartile range, and the interaction between different types of data).	
Got Friends?	Learning Task *Individual/Partner*	Validity of data, comparison of boys' versus girls' data using 5-number summary data.	MGSE7.SP.3
Travel Times to Work	Learning Task *Individual*	Comparison of travel times to work; comparing two measures of center: the mean and median when outliers are present.	MGSE7.SP.4
Thumbs on Fire Part 2	Spotlight Task *Individual/Partner*	Calculation of appropriate measures of center and measures of variability of samples; comparison of two samples and making inferences about two populations.	MGSE7.RP.1 MGSE7.RP.2 MGSE7.RP.3 MGSE7.SP.4
Emergency 911	Culminating Task *Individual Task*	Comparison of two ambulance companies; uses repeated samples of size 5. Find the mean of the sample means. Drawing conclusions from the data collected.	MGSE7.SP.1 MGSE7.SP.2 MGSE7.SP.4

SUGGESTED RESOURCES

Franklin, C., Kader, G., Mewborne, D., Moreno, J., Peck, R., Perry, M., Scheaffer, R. (2007). *Guidelines for assessment and instruction in statistics education (gaise) report: A pre-k- 12 curriculum framework.* Alexandria, VA: American Statistical Association.

Is it Valid?

Back to task table

This task seeks to help students understand the components of a survey and determine if it is bias.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.

COMMON MISCONCEPTIONS

- Students have a hard time determining when a survey is biased. This lesson seeks to help them build a concrete checklist of things to look for in finding bias.
- One sample is not always a good representation of the entire population. Many samples must be taken in order for the survey to be valid. One way to address this misconception is to compare results from one sample versus results from multiple samples.

ESSENTIAL QUESTIONS

- What are different sampling techniques used in real life?

MATERIALS

- Is It Valid? Student Sheet

GROUPING

Individual / Partner

TASK COMMENTS

Prior to the task, make sure that students are familiar with the following terms: sample and simple random sample. These terms are found in this framework. Explain to students that when sampling populations, there are factors that can affect the results of a survey. Present the following scenarios to help introduce the concepts of convenience sampling, voluntary response, and bias.

100 Atlanta residents were asked, "Who's the better quarterback, Matt Ryan or the Patriots Tom Brady?" Explain this is an example of bias. *Bias* occurs when poor sampling methods are used. The result of the sampling systematically favors certain outcomes.

The school wanted to know what is the most preferred school meal severed. Only students who bought school lunch responded. Explain this is an example of voluntary response. *Voluntary response* sampling occurs when people themselves choose to respond to a general appeal. Often,

people that volunteer to respond to questions have strong opinions, often negative ones, that may not represent the population well.

The school counselor created a survey to determine students' perspective of school. Since she was hosting the student of the month function, she decided to just survey those students. Explain this is an example of convenience sampling. *Convenience sampling* is choosing individuals that are easiest to reach.

Both convenience sampling and voluntary responses are poor sampling methods that lead to sampling bias.

TASK DESCIPTION

Is It Valid?

PART I
A **survey** is a method of collecting information. These pieces of information, numbers and facts, are called **data.**

Complete the survey questions below within your class. Listed below, you will find some possible ways for how to complete the survey.

- Have the questions on posters around the room and let each student put their name on the poster based on their personal data.
- Have the questions on a piece of paper. Give each student 5 small stickers. (The small star stickers work well.) Pass the papers around the room and have students put their sticker on their data.

DESCRIBE YOUR CLASS SURVEY

1. What color is your hair? Brown _____ Blond _____ Red _____ Black _____
2. Do you carry a cell phone? Yes_____ No _____
3. About how many text messages do you send in a day? 0-20 _____ 21-40 _____ >40 _____
4. About how much time do you spend on homework? 0-15 min _____ 16-30 min _____ 31-45 min _____ 46-60 min _____ > 60 min _____
5. What month were you born? Jan – March _____ April – June _____ July-Sept. _____Oct-Dec _____

Using the survey results discuss the following vocabulary.

6. What is the population of your survey? ***The population is the entire class. Population is the entire set of items for which data can be collected.***
7. Did your class perform a census or a survey? ***A census means you ask everyone in the population. Survey means you ask a representative sample of the population. Therefore, this is a census.***
8. How would you record your results from question 1 as a piece of data? ***Students need to record the number of students that chose each type of hair color.***
9. How would you describe your results from question 2 as a parameter for the population? ***A parameter means you describe a characteristic about the population based on your data. i.e. Most seventh grade students carry a cell phone.***

For each of the problems below, answer the questions in regard to population, samples, and surveys.

After the 2000 census, the United States Census Bureau reported that 7.4% of Georgia residents were between the ages of 10 and 14.

10. Was a parameter or a statistic reported? Explain your reasoning. ***This is an example of a statistic since it includes actual data.***

11. What is the population for this survey? ***Every person in the United States***

The manager of the Millcreek Mall wants to know the mean age of the people who shop at the mall and the stores in which they typically shop. He hires Barker Market Research Company to collect the data. Dennis works for the Barker Market Research Company and has been put in charge of collecting data for the Millcreek Mall. Dennis decides to interview 100 people one Saturday because it is the mall's busiest shopping day.

12. Is the survey a sample or a census? ***It is a sample.***

13. What is the population for this survey? ***The population is everyone who comes to the mall.***

14. Is the sample size for the survey appropriate? ***The sample size is too small for the population. The 100 people are not a representative sample of the entire population.***

PART II

There are three key things to look for when determining if a survey is biased.

15. Is the survey voluntary? ***Voluntary surveys only get respondents who feel strongly about the survey topic.***
16. Does the survey exclude any group? ***If your survey results are addressed toward a certain group or section of the population and leaves out other respondents it is biased.***
17. Is the sample size appropriate? ***If you want to make inferences about an entire city for example, yet only ask people on one street, the sample size is too small and not representative of all parts of the population.***

In the summaries below, determine if the sample taken is representative of the population without bias shown:

18. ABC Family is a television channel that targets families and young adults to view their station. ABC Family regularly posts online poll questions to their website. In 2010, ABC Family polled their viewers to ask about airing Rated "R" movies after 8pm on their channel. Almost 200,000 people responded, and 85% of them disagreed with airing Rated "R" movies.

Solution: ***Voluntary Response- ABC Family is only asking from data from their viewers, which may tend to be more conservative since it is a family targeted television station.***

19. Mrs. Jones wants to know how the 5th grade feels about recess time. Mrs. Jones labels every student in the 5th grade with a number. She then draws 50 numbers out of a hat and surveys these students. Mrs. Jones determines that 5th graders would like more recess time than they currently have.

Solution: ***This is a Simple Random Sample (SRS). Mrs. Jones gave each child in the 5th grade the same chance of being selected.***

20. The City of Smallville wants to know how its citizens feel about a new industrial park in town. Surveyors stand in the Smallville Mall from 8am-11am on a Tuesday morning and ask people their opinion. 80% of the surveyed people said they disagreed with a new industrial park.

Solution: ***This is convenience sampling. People at a mall are "easy to reach", but not every individual in Smallville has an equal chance of being chosen. For example, people that work and are not at the mall on Tuesday morning have no chance of being chosen.***

21. The National Rifle Association (NRA) took a poll on their website, www.nra.com, and asked the question, "Do you agree with the 2nd Amendment: the Right to Bear Arms"? 98% of the people surveyed said "Yes", and 2% said "No".

Solution: ***This is voluntary response. The NRA only asked people on their website, which are more likely to be "pro 2nd amendment."***

Is it Valid?

As you enter the room, please complete the five survey questions posed by your teacher.

PART I
A **survey** is a method of collecting information. These pieces of information, numbers and facts, are called **data.**

DESCRIBE YOUR CLASS SURVEY

In the space below, compile the data collected by your classmates.

1. What color is your hair? Brown _____ Blond _____ Red _____ Black _____

2. Do you carry a cell phone? Yes_____ No _____

3. About how many text messages do you send in a day? 0-20 _____ 21-40 _____ >40 _____

4. About how much time do you spend on homework? 0-15 min _____ 16-30 min _____ 31-45 min _____ 46-60 min _____ > 60 min _____

5. What month were you born? Jan – March _____ April – June _____ July-Sept. ____ _Oct-Dec _____

Using the survey results discuss the following vocabulary.

6. What is the population of your survey?

7. Did your class perform a census or a survey?

8. How would you record your results from question 1 as a piece of data?

9. How would you describe your results from question 2 as a parameter for the population?

For each of the problems below, answer the questions in regard to population, samples, and surveys.

After the 2000 census, the United States Census Bureau reported that 7.4% of Georgia residents were between the ages of 10 and 14.

10. Was a parameter or a statistic reported? Explain your reasoning.

11. What is the population for this survey?

The manager of the Millcreek Mall wants to know the mean age of the people who shop at the mall and the stores in which they typically shop. He hires Barker Market Research Company to collect the data. Dennis works for the Barker Market Research Company and has been put in charge of collecting data for the Millcreek Mall. Dennis decides to interview 100 people one Saturday because it is the mall's busiest shopping day.

12. Is the survey a sample or a census?

13. What is the population for this survey?

14. Is the sample size for the survey appropriate?

PART II

There are three key things to look for when determining if a survey is biased.

15. Is the survey voluntary?

16. Does the survey exclude any group?

17. Is the sample size appropriate?

In the summaries below, determine if the sample taken is representative of the population, without bias shown:

18. ABC Family is a television channel that targets families and young adults to view their station. ABC Family regularly posts online poll questions to their website. In 2010, ABC Family polled their viewers to ask about airing Rated "R" movies after 8pm on their channel. Almost 200,000 people responded, and 85% of them disagreed with airing Rated "R" movies.

19. Mrs. Jones wants to know how the 5^{th} grade feels about recess time. Mrs. Jones labels every student in the 5^{th} grade with a number. She then draws 50 numbers out of a hat and surveys these students. Mrs. Jones determines that 5^{th} graders would like more recess time than they currently have.

20. The City of Smallville wants to know how its citizens feel about a new industrial park in town. Surveyors stand in the Smallville Mall from 8am-11am on a Tuesday morning and ask people their opinion. 80% of the surveyed people said they disagreed with a new industrial park.

21. The National Rifle Association (NRA) took a poll on their website, www.nra.com, and asked the question, "Do you agree with the 2nd Amendment: the Right to Bear Arms"? 98% of the people surveyed said "Yes", and 2% said "No".

Snapshot

Back to task table

Adapted from ETA/Cuisenaire task Snapshot

In this task, students will determine how many tiles of each color are on the screen.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

MGSE7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples of the same size to gauge the variation in estimates or predictions.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

COMMON MISCONCEPTIONS

- Students may believe that one random sample is not representative of the entire population. Many samples must be taken in order to make an inference that is valid. By comparing the results of one random sample with the results of multiple random samples, students can correct this misconception.

ESSENTIAL QUESTIONS

- How can random samples be used to make predictions about populations?
- How are proportions used to estimate information about populations?

MATERIALS

- Snapshot
  flipchart https://drive.google.com/file/d/0B0S_r7_2OlzNSEoxVnU0OVh3QlE/edit?usp=sharing
  (works with Promethean, SMART board and Mimio)
- Snap cubes or linking cubes or centimeter cubes

GROUPING

Small Groups

TASK DESCRIPTION

Open the Snapshot flipchart. Ensure the Spotlight tool has been selected (the screen should be black with a circular view of the images hidden). https://drive.google.com/file/d/0B0S_r7_2OlzNSEoxVnU0OVh3QlE/edit?usp=sharing

A snapshot freezes a moment in time. A series of snapshots becomes a picture record of an event. For this task, you'll use Snap Cubes within a flipchart to create snapshots.

• There are 22 Snap Cubes in the Snapshot flipchart.
• Your teacher will scan the spotlight until a Snap Cube appears in the opening. Record the color of the cube. Think of each sample as a snapshot of the contents of the flipchart.
• Keep taking and recording "snapshots" until you think you have enough data to make a prediction.

Use your data to predict how many of each color there are in the chart. Be ready to show your data and explain how you arrived at your prediction.

There are 6 red cubes, 2 white cubes, 5 yellow cubes, 4 green cubes, and 5 blue cubes.

In your journal, write whether or not your prediction was reasonably accurate. What would you do differently next time, if anything, to make a closer prediction?

Snapshot

A snapshot freezes a moment in time. A series of snapshots becomes a picture record of an event. For this task, you'll use Snap Cubes within a flipchart to create snapshots.

• There are 22 Snap Cubes in the Snapshot flipchart.
• Your teacher will scan the spotlight until a Snap Cube appears in the opening. Record the color of the cube. Think of each sample as a snapshot of the contents of the flipchart.
• Keep taking and recording "snapshots" until you think you have enough data to make a prediction.

Use your data to predict how many of each color there are in the chart. Be ready to show your data and explain how you arrived at your prediction.

Write whether or not your prediction was reasonably accurate. What would you do differently next time, if anything, to make a closer prediction?

Candy Populations

Back to task table

Adapted from Birdville County, Texas Probability and Statistics Unit
http://schools.birdville.k12.tx.us/cms/lib2/TX01000797/Centricity/Domain/852/LAbs%20with%20MnMs.pdf

In this task, students will draw inferences about a population of M&Ms based upon random samples of M&Ms using proportional reasoning developed in unit three.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

MGSE7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples of the same size to gauge the variation in estimates or predictions.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

In order for students to be successful, the following skills and concepts need to be maintained:

- Change a fraction to a percentage.
- Understand how to construct a proportional relationship and identify how parts of proportions are related
- Solve a proportion for an unknown variable
- Use multiplication and division or rational numbers to solve real-world problems

*It is very important that students *already* know how to set up a proportion in order to solve this task. This is not a learning task, but a performance task that helps to further develop student reasoning about samples and populations not proportions.

COMMON MISCONCEPTIONS

- *Students may believe that one random sample is not representative of the entire population. Many samples must be taken in order to make an inference that is valid. By comparing the results of one random sample with the results of multiple random samples, students can correct this misconception.*

ESSENTIAL QUESTIONS

- How can random samples be used to make predictions about populations?
- How are proportions used to estimate information about populations?
- How do I determine an appropriate sample size?

MATERIALS

- M&Ms *Trial Sized may be easier
- Small plastic bags
- Paper plates or paper towels
- Task Sheet

GROUPING

Small Groups

TASK DESCRIPTION

Begin the lesson with a Number Talk. Number talks are a great way for students to use mental math to solve and explain a variety of math problems. A Number Talk is a short, ongoing daily routine that provides students with meaningful ongoing practice with computation. Number Talks should be structured as short sessions alongside (but not necessarily directly related to) the ongoing math curriculum. It is important to keep Number Talks short, as they are not intended to replace current curriculum or take up the majority of the time spent on mathematics.

In fact, teachers need to spend only 5 to 15 minutes on Number Talks. Number Talks are most effective when done every day. As previously stated, the primary goal of Number Talks is computational fluency. Students develop computational fluency while thinking and reasoning like mathematicians. When they share their strategies with others, they learn to clarify and express their thinking, thereby developing mathematical language. This in turn serves them well when they are asked to express their mathematical processes in writing. In order for students to become computationally fluent, they need to know particular mathematical concepts that go beyond what is required to memorize basic facts or procedures.

All Number Talks follow a basic six-step format. The format is always the same, but the problems and models used will differ for each number talk.

1. Teacher presents the problem. Problems are presented in a word problem or a written algorithm.

2. Students figure out the answer. Students are given time to figure out the answer. To make sure students have the time they need, the teacher asks them to give a “thumbs-up” when they have determined their answer. The thumbs up signal, given at chest level, is unobtrusive- a message to the teacher, not the other students.

3. Students share their answers. Four or five students volunteer to share their answers and the teacher records them on the board.

4. Students share their thinking. Three or four students volunteer to share how they got their answers. (Occasionally, students are asked to share with the person(s) sitting next to them.) The teacher records the student's thinking.

5. The class agrees on the "real" answer for the problem. The answer that together the class determines is the right answer is presented as one would the results of an experiment. The answer a student comes up with initially is considered a conjecture. Models and/or the logic of the explanation may help a student see where their thinking went wrong, may help them identify a step they left out, or clarify a point of confusion. There should be a sense of confirmation or clarity rather than a feeling that each problem is a test to see who is right and who is wrong. A student who is still unconvinced of an answer should be encouraged to keep thinking and to keep trying to understand. For some students, it may take one more experience for them to understand what is happening with the numbers and for others it may be out of reach for some time. The mantra should be, "If you are not sure or it doesn't make sense yet, keep thinking."
6. The steps are repeated for additional problems.

Similar to other procedures in your classroom, there are several elements that must be in place to ensure students get the most from their Number Talk experiences. These elements are:

1. A safe environment
2. Problems of various levels of difficulty that can be solved in a variety of way
3. Concrete models
4. Opportunities to think first and then check
5. Interaction
6. Self-correction

For this Number Talk, begin with the following problem, “What is 0.75 as a percent?” Record the problem on the far left side of the board. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the “thumbs up” signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you. It is important to allow students ownership of their thinking.

Record, “What is ¾ as a percent?” on the board next to the previous problem. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the “thumbs up” signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

Record, “What is 0.5 as a percent?” on the board towards the right. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the “thumbs up” signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

Record, “What is 15/30 as a percent?” on the board towards the far right. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the “thumbs up” signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

At the end of the Number Talk, discuss the strategies used to find the answers. The purpose of this number talk is to discuss and develop strategies for converting decimals and fractions to percent. Some of the strategies students may use are setting up proportions and finding the unknown, multiplying the decimal with 100, use properties of place value to determine the percent is 100 times greater than the number and changing the fraction to a decimal and multiplying by 100. Talk with the students about which strategy was most efficient (quick, easy and accurate).

Allow for a maximum of 15 minutes to conduct the Number Talk before moving into the lesson.

To begin this task:

1. Give each student a small bag of M&Ms with approximately 10-15 M&Ms in each.
   *Note: Each bag needs to represent a random sample- teachers need to make sure that M&Ms are not assorted by color. Each group should have a total of 50-60 M&Ms. Trial sized M&Ms may be the best sample because they are already pre-packaged.
2. Pass out additional materials and task sheets

M&M Populations

NOTE: The following solutions are based upon a group of five sets of 10 M&Ms. Numbers will vary depending on sample sizes that teachers create for their students.

1. Count the number of EACH color of M&Ms and record your answers in a frequency table. Make sure to label your frequency table clearly.

   Solution:
   Students may choose a variety of different frequency tables, including a tally chart or line plot.

2. Using the results from your group, fill in the following chart and find the number of M&Ms for each person in your group and the total number of M&Ms.
   Find the percentage of each color of M&Ms for your sample.

   Suppose that a group of students received the following samples:

M&M Color	Number of M&Ms in Sample # 1	Number of M&Ms in Sample # 2	Number of M&Ms in Sample # 3	Number of M&Ms in Sample # 4	Number of M&Ms in Sample # 5	Number of M&Ms in Sample # 6	Total Number of M&Ms in ALL Samples	% of Each Color of M&Ms
Red	*2*	*1*	*3*	*3*	*2*		*11*	$\frac{11}{50}$=*22%*
Orange	*1*	*1*	*2*	*1*	*1*		*6*	$\frac{6}{50}$=*12%*
Yellow	*1*	*1*	*1*	*2*	*1*		*6*	$\frac{6}{50}$=*12%*

Green	*1*	*2*	*1*	*1*	*2*		*7*	$\frac{7}{50}$=*14%*
Blue	*1*	*3*	*2*	*1*	*1*		*8*	$\frac{8}{50}$=*16%*
Brown	*4*	*2*	*1*	*2*	*3*		*12*	$\frac{12}{50}$=*24%*
Total Number of M&Ms in Sample	*10*	*10*	*10*	*10*	*10*		*50*	*100%*

Answer the following questions about your sample:

3. Is your sample random? Explain your answer.
   Yes, our teacher gave use the M&Ms in a random assortment.

4. Do you think your sample could predict the number of M&Ms in a larger bag? Why or why not?
   Yes, because we have a small, random sample that we can use to make predictions for a larger bag of M&Ms, as long as they are the same kind of M&Ms.
   ****Teachers should point out that samples need to be representative. For example, it does not make sense to use Mini-M&Ms to estimate for a large bag of Peanut M&Ms.***

5. Do you think that the percentages of each color of M&Ms are the same as other groups? Why or why not?
   No. Other groups received different samples of M&Ms. Percents may be close to one another but they will not be the exact same.

6. Using your data values, estimate the number of M&Ms for EACH color for a bag of 1000 candies. (Hint: Use a proportion)

   Students will use proportional reasoning to estimate the different percentages of each M&M color in a given population.
   It is important to help students recognize that this type of reasoning is ONLY truly valid when using valid, random samples.

NOTE: The following solutions are taken from the above samples. Answers will vary as students use varying sample sizes and amounts

Color	**Estimate for Number of M&Ms in a Population of 1000 M&Ms**
Red	$\frac{11}{50} = \frac{x}{1000}$ $50x = 11(1000)$ $50x = 11000$ $x = \frac{11000}{50} = 220\ M\&Ms$
Orange	$\frac{6}{50} = \frac{x}{1000}$ $50x = 6(1000)$ $50x = 6000$ $x = \frac{6000}{50} = 120\ M\&Ms$
Yellow	$\frac{6}{50} = \frac{x}{1000}$ $50x = 6(1000)$ $50x = 6000$ $x = \frac{6000}{50} = 120\ M\&Ms$
Green	$\frac{7}{50} = \frac{x}{1000}$ $50x = 7(1000)$ $50x = 7000$ $x = \frac{7000}{50} = 140\ M\&Ms$
Blue	$\frac{8}{50} = \frac{x}{1000}$ $50x = 8(1000)$ $50x = 8000$ $x = \frac{8000}{50} = 160\ M\&Ms$
Brown	$\frac{12}{50} = \frac{x}{1000}$ $50x = 12(1000)$ $50x = 12000$ $x = \frac{12000}{50} = 240\ M\&Ms$

DIFFERENTIATION

Extension

- Have students create an accurate circle (pie) graph to represents the percentages of each sample and compare them between each sample group to discuss random sampling
- Students can create a circle graph in Excel using data entries found in this task, and also use the same method to create circle graphs for the entire population of M&Ms to make comparisons

Intervention

- Small groups can be given examples to help guide them through this performance task
- Sample sizes and population sizes can be adjusted to help guide students through this task

Candy Populations

1. Count the number of EACH color of candies and record your answers in a frequency table. Make sure to label your frequency table clearly.

2. Using the results from your group, fill in the following chart and find the number of M&Ms for each person in your group and the total number of M&Ms.
   Find the percentage of each color of M&Ms for your sample.

Candy Color	**Number of candies in Sample # 1**	**Number of candies in Sample # 2**	**Number of candies in Sample # 3**	**Number of candies in Sample # 4**	**Number of candies in Sample # 5**	**Number of candies in Sample # 6**	**Total Number of candies in ALL Samples**	**% of Each Color of candies**
Red								
Orange								
Yellow								
Green								
Blue								
Brown								
Total Number of M&Ms in Sample								

Answer the following questions about your sample:

3. Is your sample random? Explain your answer.

4. Do you think your sample could predict the number of M&Ms in a larger bag? Why or why not?

5. Do you think that the percentages of each color of M&Ms are the same as other groups? Why or why not?

6. Using your data values, estimate the number of M&Ms for EACH color for a bag of 1000 candies. Record your data in the table below.

Color	**Estimate for Number of M&Ms in a Population of 1000 M&Ms**
Red	
Orange	
Yellow	
Green	
Blue	
Brown	

Predicting Population

Back to task table

(This task has been adapted from http://southwest.mpls.k12.mn.us/uploads/Goldfish_Lab.cwk__WP_.pdf)

In this task, students will use populations, samples, and proportions in order to make predictions about total population size.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population. Understand that random sampling tends to produce representative samples and support valid inferences.

MGSE7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples of the same size to gauge the variation in estimates or predictions.

STANDARDS OF MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.

COMMON MISCONCEPTIONS

Students struggle with determining how many items should be in a sample space in order for it to reflect the given population.

BACKGROUND KNOWLEDGE

In a previous unit, students studied how to set up and solve proportions. They will need this information in order to solve and make predictions.

ESSENTIAL QUESTIONS

- How do I determine an appropriate sample size?
- How can random samples be used to make predictions about populations?
- How are proportions used to estimate information about populations?

MATERIALS

- Predicting Populations Student Sheet
- Paper bag with a 40 fish crackers in it
- Black Marker

GROUPING

Individual, Partner, or Group

TASK DESCRIPTION

If the population size of a group is unknown, the sample size can be used to make predictions and calculate this number. When predicting deer population, a group of deer are captured, marked, and then distributed back into the population. Later, a sample is taken and the number of marked deer is compared to the total number of deer in the sample. A proportion can be set up to find the ratio of marked deer to total sample deer.

For example:

A forest has too many deer for us to count. We take a group of 100 deer and mark them then distribute them evenly throughout the population. Several days pass, then we take a sample of 120 deer and find that 20 of them are marked. We can set up a proportion to calculate the total population size.

i.e. $\frac{20}{120} = \frac{100}{x}$ x = 100(120) / 20

So, our estimate is there are 600 deer in our population.

This is an ideal scenario. Based on the size of the land, you may need a larger starting group to mark. Otherwise, you may take a sample and not have any tagged deer. Likewise, if you sample size of group two is not large enough it might not reflect the population.

Goldfish Lab

You have a bag with fish crackers in it. We are going to "tag" a sample of the fish and make a prediction about the total population of fish found in the bag.

1. Remove 10 crackers from the bag.
2. Tag them by marking on them with a marker.
3. Put the fish back in the bag and shake them up.
4. Remove 20 fish crackers.
5. Set up a proportion and make a prediction for how many fish are in the bag total.
   $i.e. \frac{4\ tagged}{20\ in\ the\ sample} = \frac{10}{x}; x = 50$ *is the estimated population*

We will fill in the following chart based on the data from each group.

CLASS DATA TABLE

Group Number	Number of tagged fish in the sample	Total number in the sample	Total number of tagged individuals in the population	Total Estimated Population
One				
Two				
Three				
Four				
Five				
Six				
Seven				
Eight				

1. What is the average estimated population for your class? ***Find the mean of the last column.***
2. Count the number of fish in the bag. What was the actual population size? *40*

3. Is this a census or a survey? Justify your response. ***This is a survey since you take a part of the population.***
4. What is your percent of error based on the estimated population and actual population?

$$\left(\frac{difference\ between\ actual\ and\ predicted\ population}{actual\ population}\right)100 = \text{percent of error}$$

5. Does this method seem reliable for wildlife population? Explain.

   Answers May Vary. Students may have differing opinions.

6. If there is a population of 800 deer, what would be a good sample size? Justify your answer. ***25 – 40% of the population makes a good sample size. So you would need about 200 deer in your sample group.***

Goldfish Lab

You have a bag with fish crackers in it. We are going to "tag" a sample of the fish and make a prediction about the total population of fish found in the bag.

1. Remove 10 crackers from the bag.
2. Tag them by marking on them with a marker.
3. Put the fish back in the bag and shake them up.
4. Remove 20 fish crackers.
5. Set up a proportion and make a prediction for how many fish are in the bag?

We will fill in the following chart based on the data from each group.

CLASS DATA TABLE

Group Number	Number of tagged fish in the sample	Total number in the sample	Total number of tagged individuals in the population	Total Estimated Population
One				
Two				
Three				
Four				
Five				
Six				
Seven				
Eight				

1. What is the average estimated population for your class?

2. Count the number of fish in the bag. What was the actual population size?

3. Is this a census or a survey? Justify your response.

4. What is your percent of error based on the estimated population and actual population?

5. Does this method seem reliable for wildlife population? Explain.

6. If there is a population of 800 deer, what would be a good sample size? Justify your answer.

Counting Trees

Back to task table

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=1148

(Problem Solving Task)
This lesson is intended to help you assess how well students are able to:

- Solve simple problems involving ratio and direct proportion
- Choose an appropriate sampling method
- Collect data and organize it using a frequency table

STANDARDS FOR MATHEMATICAL CONTENT

Apply and extend previous understandings of measurement and interpreting data.

MGSE7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

MGSE7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

MGSE7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the medians by expressing it as a multiple of the interquartile range.

MGSE7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

STANDARDS OF MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

ESSENTIAL QUESTION

- How do scientists make estimations about a population size using a representative sample?
- How can random samples be used to make predictions about populations?

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Counting Trees*, is a Mathematics Assessment Project Assessment Task that can be found at the website: http://www.map.mathshell.org/materials/tasks.php?taskid=386&subpage=expert

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=1148

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=1149

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Human Box Plot

Back to task table

Students will physically construct a box plot using given data.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the medians by expressing it as a multiple of the interquartile range.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision

ESSENTIAL QUESTIONS

- How does the data describe its center, spread, and representation of the population?
- How do I display data on a number line?

MATERIALS

- String
- Index cards

GROUPING

Whole Group

TASK COMMENTS:

This task serves as a mini-lesson on constructing a box plot. The following activity can be used as a mini lesson for the Shakespeare vs. Harry Potter task.

TASK DESCRIPTION

The following numbers represent the ages in months of a class of 7th graders.

Boys	
142	125
134	147
137	131
139	129

Girls	
140	131
129	128
141	131
134	

Instruct students to create a stem and leaf plot for all students using the data above.

All students

12	5, 8, 9, 9
13	1,1,1,4,4,7,9
14	0,1,2,7

Assign students in the class a number from the data above. Have students line up in order from least to greatest. Have the students in the audience identify the median of the data (*134*). Label this data point the median. Looking only at the lower half of the data, have students determine the median (*129*). Inform students this is called the lower quartile or Q1, label this on the human display. Looking only at the upper half of the data, have students determine the median (*140*). Inform students this is called the upper quartile or Q3, label this on the human display.
Using string, create a box around Q1, median and Q3. Notate the two extreme data points (*125*, *147*) as the minimum and maximum. Using two pieces of string, stretch one piece from Q1 to the minimum and the other string from Q3 to the maximum.

Ask the students in the audience to record the human display of a box plot in their notebooks. Be sure the label the key components of the plot. Have students return to their seats, and reiterate the steps for creating the box plot by reviewing the steps used in the demonstration.

Instruct students to complete the 5-number summary using the recreated box plot.

Minimum	***125***
Q1	***129***
Median	***134***
Q3	***140***
Maximum	***147***

Shakespeare vs. Harry Potter

Back to task table

Students will analyze text from two periods of time to determine which author used longer words. Students will randomly sample given text from two pieces of literature

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the medians by expressing it as a multiple of the interquartile range.

MGSE7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision

ESSENTIAL QUESTIONS

- How does the data describe its center, spread and representation of the population?
- What are ways to summarize numerical data set?

MATERIALS

- Shakespeare vs. Harry Potter Task Sheet
- Number cube
- Graphing Calculator (optional)

GROUPING

Small Group

TASK COMMENTS:

Prior to the task, students may need a mini-lesson about the following concepts: box plots, dot plots, 5-number summary and interquartile range.

TASK DESCRIPTION

Many skeptics feel that there has been a "dumbing down" of America's youth of the past decades. To determine if there is any truth to this claim, we will compare two pieces of literature: Shakespeare's *Macbeth* and JK Rowling's *Harry Potter and the Chamber of Secrets*.

Is there a difference in the length of the words used in a Shakespeare play compared to a Harry Potter book? Today you will sample words from both pieces of literature to determine who used longer words.

Below are excerpts from a Shakespeare Novel and a Harry Potter book.
Follow the steps below to determine which piece of literature uses longer words.

1. Roll a number cube once for each line of *Macbeth* below. Move to the word that corresponds to the number you roll (1 = first word in the line, 2 = second word in the line, etc.).
2. Count the letters in that word, and record the number in the table below in the "Letter Count" columns.

1. *Is this a dagger which I see before me,*
2. *The handle toward my hand? Come, let me clutch thee!*
3. *I have thee not, and yet I see thee still.*
4. *Art thou not, fatal vision, sensible*
5. *To feeling as to sight, or art thou but*
6. *A dagger of the mind, a false creation,*
7. *Proceeding from the heat oppressed brain?*
8. *I see thee yet, in form as palpable*
9. *As this which now I draw.*
10. *Though marshal'st me the way I was going;*
11. *And such an instrument I was to use.*
12. *Mine eyes are made the fools o' th' other senses,*
13. *Or else worth all the rest. I see thee still;*
14. *And on thy blade and dugeon gouts of blood,*
15. *Which was not so before. There's no such thing.*

Sample:

Line Number	Letter Count	Line Number	Letter Count
1	*6*	9	
2	*9*	10	
3	*12*	11	
4	*10*	12	
5	*4*	13	
6	*7*	14	
7		15	
8			

Comment: student data will vary because they are using number cubes to generate the word that they count letters for in each line.

3. Now, roll the number cube to see which words you will count in the Harry Potter excerpt. You will count the letters in two words from each line. Roll the die and move to the word that corresponds to that number. After counting the number of letters in that word, roll the dice again and move to another word in the same line. Then, move to the next line and repeat the process for each of the lines in the passage. Record your numbers in the chart that follows in the "Letter Count" columns.

1. *October arrived, spreading a damp chill over the grounds and into the*
2. *castle. Madam Pomfrey, the nurse, was kept busy by a sudden*
3. *spate of colds among the staff and students. Her Pepperup potion*
4. *worked instantly, though it left the drinker smoking at the ears for*
5. *several hours afterward. Ginny Weasley, who had been looking*
6. *pale, was bullied into take some by Percey. The steam pouring from*
7. *under her vivid hair gave the impression that her whole head was on fire.*

Sample:

Line Number	Letter Count Word 1	Letter Count Word 2
1	*1*	*7*
2	*4*	
3	*13*	
4	*7*	
5	*9*	
6	*5*	
7	*7*	

***Comment*: student data will vary because they are using number cubes to generate the word that they count letters for in each line.**

4. Find the mean of the Shakespeare word sample and the Harry Potter word sample. Do this by adding the letter counts for both passages, separately. Then divide the sum for each passage by the total number of words (15 for Shakespeare; 14 for Harry Potter).

Shakespeare mean ($\bar{x}$) =_____ Harry Potter mean ($\bar{x}$) =_____

***Comment*:**
There may be more efficient ways of finding the mean (instead of the method below).

Solution: (sample)
Shakespeare:

$$\frac{\textit{total letters counted}}{\textit{\# of words counted}} = \frac{6+9+12+10+4+7+\cdots}{15} = \frac{120}{15}$$
$$= 8\ \textit{letters/word}$$

Harry Potter:

$$\frac{\textit{total letters counted}}{\textit{\# of words counted}} = \frac{1+4+13+7+9+5+7+7+\cdots}{14}$$
$$= \frac{126}{14} = 9\ \textit{letters/word}$$

5. Find the five number summary using the data you found for Shakespeare and Harry Potter. Enter your data into the table below:

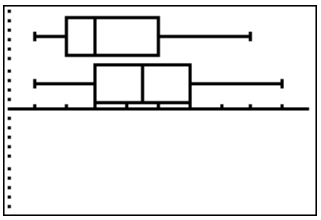
Sample:

Shakespeare		**Harry Potter**	
Minimum	*1*	Minimum	*1*
Q1	*2*	Q1	*3*
Median	*3*	Median	*4.5*
Q3	*5*	Q3	*6*
Maximum	*8*	Maximum	*9*

6. Create a box plot for the word counts you found for Shakespeare and Harry Potter. Make sure to label your number line:

Solution:

Stacked Comparison: Shakespeare on top, Harry Potter below:



7. Looking at the box plots and the mean, would you agree that there has been a "dumbing down" of America's youth over the past decades? Support your answer with numerical data you found in steps #4-8.

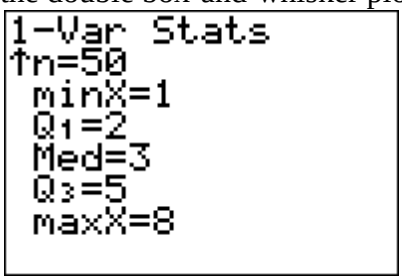
Solution: ***The overall mean number of words was higher for Harry Potter (4.68) as compared to Shakespeare's mean of 3.54. Each piece of literature shared the minimum value of 1, but Harry Potter's maximum value was higher than Shakespeare. With is evidence, it is clear that there has not been a "dumbing down" of America's youth.***

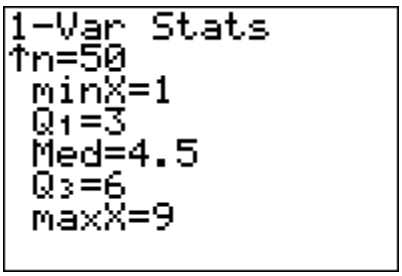
8. Do you believe the comparison above could help you conclude the word counts for ***ALL*** Harry Potter and Shakespeare Literature? Why or why not?

Solution: ***Since each Harry Potter novel and each piece of literature by Shakespeare varies, it is not wise to conclude that the words counts would be similar for other types of literature written by these authors. A sample taken from another book or poem may yield different answers.***

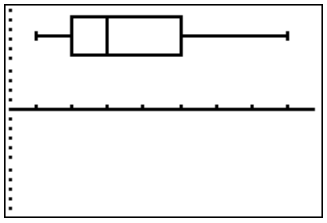
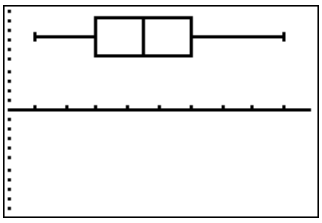
DIFFERENTIATION

- **Extension**
  If available, students can use graphing calculators to input the five number summary and create the double box-and-whisker plot.





- **Intervention:** Students can create individual box-and-whisker plots for each piece of literature before creating the double box-and-whisker plot. These images are from a graphing calculator; students should add numbers to their number lines when they create their own box-and-whisker plots.

Shakespeare:  ***Harry Potter:*** 

- Two different literature pieces could be taken that have easier to comprehend language. Two suggestions that would tie into seventh grade language arts are *While the World Watched* by Carolyn Maull McKinstry and *The Watsons Go to Birmingham-1963* by Christopher Paul Curtis.

Shakespeare vs. Harry Potter

Many skeptics feel that there has been a "dumbing down" of America's youth of the past decades. To determine if there is any truth to this claim, we will compare two pieces of literature: Shakespeare's *Macbeth* and JK Rowling's *Harry Potter and the Chamber of Secrets*.

Is there a difference in the length of the words used in a Shakespeare play compared to a Harry Potter book? Today you will sample words from both pieces of literature to determine who used longer words.

Below are excerpts from a Shakespeare Novel and a Harry Potter book.
Follow the steps below to determine which piece of literature uses longer words.

1. Roll a number cube once for each line of *Macbeth* below. Move to the word that corresponds to the number you roll (1 = first word in the line, 2 = second word in the line, etc.).
2. Count the letters in that word, and record the number in the table below in the "Letter Count" columns.

1. *Is this a dagger which I see before me,*
2. *The handle toward my hand? Come, let me clutch thee!*
3. *I have thee not, and yet I see thee still.*
4. *Art thou not, fatal vision, sensible*
5. *To feeling as to sight, or art thou but*
6. *A dagger of the mind, a false creation,*
7. *Proceeding from the heat oppressed brain?*
8. *I see thee yet, in form as palpable*
9. *As this which now I draw.*
10. *Though marshal'st me the way I was going;*
11. *And such an instrument I was to use.*
12. *Mine eyes are made the fools o' th' other senses,*
13. *Or else worth all the rest. I see thee still;*
14. *And on thy blade and dugeon gouts of blood,*
15. *Which was not so before. There's no such thing.*

Shakespeare:

Line Number	Letter Count	Line Number	Letter Count
1		9	
2		10	
3		11	
4		12	
5		13	
6		14	
7		15	
8			

3. Now, roll the number cube to see which words you will count in the Harry Potter excerpt. You will count the letters in two words from each line. Roll the die and move to the word that corresponds to that number. After counting the number of letters in that word, roll the dice again and move to another word in the same line. Then, move to the next line and repeat the process for each of the lines in the passage. Record your numbers in the chart that follows in the "Letter Count" columns.

1. *October arrived, spreading a damp chill over the grounds and into the*
2. *castle. Madam Pomfrey, the nurse, was kept busy by a sudden*
3. *spate of colds among the staff and students. Her Pepperup potion*
4. *worked instantly, though it left the drinker smoking at the ears for*
5. *several hours afterward. Ginny Weasley, who had been looking*
6. *pale, was bullied into take some by Percey. The steam pouring from*
7. *under her vivid hair gave the impression that her whole head was on fire.*

Harry Potter:

Line Number	Letter Count Word 1	Letter Count Word 2
1		
2		
3		
4		
5		
6		
7		

4. Find the mean ($\bar{x}$) of the Shakespeare word sample and the Harry Potter word sample. Do this by adding the letter counts for both passages, separately. Then divide the sum for each passage by the total number of words (15 for Shakespeare; 14 for Harry Potter).

Shakespeare mean ($\bar{x}$) =_____ Harry Potter mean ($\bar{x}$) =_____

5. Find the five number summary using the data you found for Shakespeare and Harry Potter. Enter your data into the table below:

Shakespeare		**Harry Potter**	
Minimum		Minimum	
Q1		Q1	
Median		Median	
Q3		Q3	
Maximum		Maximum	

6.Create a box plot for the word counts you found for Shakespeare and Harry Potter. Make sure to label your number line:

7.Looking at the box plots and the mean, would you agree that there has been a "dumbing down" of America's youth over the past decades? Support your answer with numerical data you found in steps #4-

8. Do you believe the comparison above could help you conclude the word counts for ***ALL*** Harry Potter and Shakespeare Literature? Why or why not?

Box and Whisker Plots – FAL

Back to task table

Source: Georgia Mathematics Design Collaborative
This lesson is intended to help you assess how well students are able to:

- Utilize what they know about box and whisker plots to evaluate sets of data
- Assimilate multiple sets of data related to a single entity, and combine this data in order to make value judgments about that entity, particularly when the information found in the data seems to be contradictory
- Compare entities and make decisions about "good," "better," and "best" based upon distribution of summary data (medians, quartiles, interquartile range, and the interaction between different types of data).

STANDARDS FOR MATHEMATICAL CONTENT:

MGSE7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the medians by expressing it as a multiple of the interquartile range.

MGSE7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
7. Look for and make use of structure

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, Box and Whisker Plots, is a Formative Assessment Lesson (FAL) that can be found at: **http://ccgpsmathematics6-8.wikispaces.com/Georgia+Mathematics+Design+Collaborative+Formative+Assessment+Lessons**

Got Friends?

Back to task table

Students will use their knowledge of graphs, mean, median, and interquartile range to compare two sets of data.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the medians by expressing it as a multiple of the interquartile range.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

COMMON MISCONCEPTIONS

Students have difficulties distinguishing between measures of center and measures of variability. It is essential to focus questions on the context of the situation and seeing what can be learned or inferred from the data. In addition, the questions should focus on the key ideas of statistics, such as variability, center of the data, and shape of the data.

ESSENTIAL QUESTIONS

- How does the data describe its center, spread and representation of the population?
- What is the difference between the measure of center and measure of variation?

MATERIALS

- Student Task Sheet- Got Friends?
- Graphing Calculator (optional)

GROUPING

Individual/Partner

TASK COMMENTS

Prior to completing the task, teachers may need to review box plots, dot plots, interquartile range, measures of center, and measures of variability.

TASK DESCRIPTION

TE Learning Task: Got Friends?

Is there a difference between the number of programmed numbers in the number of programmed telephone numbers in girls' cell phones and the number in boys' cell phones?

Do you think there is a difference? Why or why not?

Answer may vary and will be based on student opinions.

When Mrs. Causey, an AP Statistics teacher at Olviedo High School in Seminole County, Florida, polled her students, she got the following data:

Males	5	20	26	40	46	47	49	50	51
51	56	57	60	61	68	71	72	72	73
74	75	82	82	84	86	97	100	100	104
104	106	124	171	205	207	232	360		

Females	20	46	50	58	62	65	70	72	72
80	86	87	88	90	92	94	94	109	112
114	116	122	125	129	137	137	138	142	142
149	163	170	186	199	204	249			

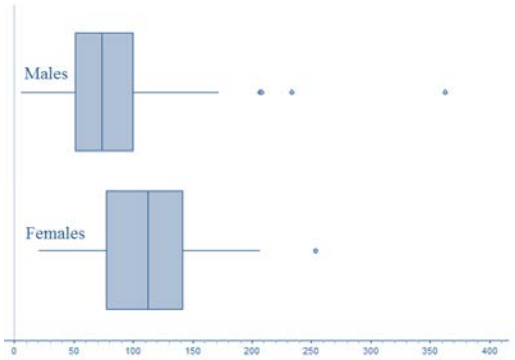
1. Find the 5 number summary for both boys and girls:

Solution:

Males		**Females**	
Minimum	***5***	Minimum	***12***
Q1	***51***	Q1	***72***
Median	***73***	Median	***101.5***
Q3	***102***	Q3	***140***
Maximum	***360***	Maximum	***249***

3. Create side-by-side (stacked) box plots for the two sets of data above.

If there are not graphing calculators available, have the students find the statistics by hand that are required for the 5 Number Summary. (using the median)



Solution:

The Males are represented by the box plot on the top, females on the bottom.

4. Compare the box plots above. Do you notice a difference in their shape, center, and spread?

Solution:

Shape: the female's box plot is more symmetric, with the median lying approximately in the middle of the box. The whiskers also appear to be approximately equal in length. This leads to the conclusion that the female distribution is symmetric. There appears to be no outliers. The male distribution is much more skewed. The right whisker is very long, which means it is reaching out to a potential outlier in the male data (360).

Center (median): The center for the males is 73, while the center for the females is 101.5. This means that females seem to have more numbers programmed in their phone than males.

Spread: The spread of the males is much greater than for females. This is due to extreme values on the upper end of the male data.

5. Which gender group has the greater interquartile range? Does this align with the group with the most programmed phone numbers?

The IQR for the males is 51, the IQR for the females is 68.

6. It is important that you have "data integrity". For example, it is important that data be reported accurately and truthfully. Do you think that this is the case here? Do you see any suspicious observations?

Solution:

There appears to be a suspicious value in the male data. 360 programmed numbers seems to be well outside the range of the other values.

7. Can you think of any reason someone might make up a response or "stretch the truth" in reporting his or her number of programmed telephone numbers?

Solution:
Students may feel the need to exaggerate their answers due to peer pressure of classmates. Good sampling techniques, such as writing out the numerical amount, instead of calling it out in front of the class, may help produce more accurate results.

8. If you DO see a difference between the two groups, can you suggest a possible reason for that difference?

Solution:
Females tend to have more numbers in their phone. Females may be more likely to talk on the phone, which could lead to more programmed numbers in their phone. The parents of females may have given them a cell phone earlier to use for safety reasons.

9. Do you think that a study of cell phone programmed numbers for a 7th grade math class would yield similar results? Why or why not?

Solution:
No. The data for 7th grade may yield the same shape of distribution, but the amount of programmed numbers may increase with the grade level.

DIFFERENTIATION

Extension

- Ask students to generate data points that will change the box and whisker plots significantly and explain why such changes would affect the numbers for the box and whisker plots.

Intervention

- Work through the 5 Number Summary - boys or girls with the students and let them do the other 5 Number Summary by themselves. The 5 Number Summary was introduced in 6th grade.

Got Friends?

Is there a difference between the number of programmed numbers in the number of programmed telephone numbers in girls' cell phones and the number in boys' cell phones?

1. Do you think there is a difference? Why or why not?

When Mrs. Causey, an AP Statistics teacher at Olviedo High School in Seminole County, Florida, polled her students, she got the following data:

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51	56	57	60	61	68	71	72	72	73
74	75	82	82	84	86	97	100	100	104
104	106	124	171	205	207	232	360		

Females	20	46	50	58	62	65	70	72	72
80	86	87	88	90	92	94	94	109	112
114	116	122	125	129	137	137	138	142	142
149	163	170	186	199	204	249			

2. Find the 5 number summary for both boys and girls:

Males		**Females**	
Minimum		Minimum	
Q1		Q1	
Median		Median	
Q3		Q3	
Maximum		Maximum	

3. Create side-by-side (stacked) box plots for the two sets of data above.

4. Compare the box plots above. Do you notice a difference in their shape, center, and spread?

5. Which gender group has the greater interquartile range? Does this align with the group with the most programmed phone numbers?

6. It is important that you have "data integrity"? For example, it is important that data be reported accurately and truthfully. Do you think that this is the case here? Do you see any suspicious observations?

7. Can you think of any reason someone might make up a response or "stretch the truth" in reporting his or her number of programmed telephone numbers?

8. If you DO see a difference between the two groups, can you suggest a possible reason for that difference?

9. Do you think that a study of cell phone programmed numbers for a 7^{th} grade math class would yield similar results? Why or why not?

Travel Times to Work

Back to task table

Students will calculate the measures of center and the measures of variability to compare two sets of data.

STANDARDS OF MATHEMATICAL CONTENT

MGSE7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the medians by expressing it as a multiple of the interquartile range.

MGSE7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

COMMON MISCONCEPTIONS

- *Students do not understand the difference between measures of center and measures of variability*
- *Students believe that the mean is always the best, even though the median can be a better measure*

ESSENTIAL QUESTIONS

- How does the data describe its center, spread and representation of the population?
- What is the difference between the measure of center and measure of variation?

MATERIALS

- Student Task Sheet
- Calculators (Optional) *Use to efficiently calculate mean

GROUPING

Individual / Partners

TASK COMMENTS

Sample scenarios or problems may need to be discussed for students to make comparisons about the two measures and determine which set of measures is best to use.

TASK DESCRIPTION

Travel Times to Work

How long does it take you to get from home to school?

Here are the travel times from home to work in minutes for 15 workers in North Carolina, chosen at random by the Census Bureau:

5	10	10	10	10	12	15	20	20	25	30	30	40	40	60

1. Find the mean of the North Carolina travel times.

Solution:

$$mean = \frac{5 + 10 + 10 + 10 + 10 + 12 + 15 + 20 + 20 + 25 + 30 + 30 + 40 + 40 + 60}{15} = \frac{337}{15} = 22.47\ minutes$$

Here are the travel times in minutes of 19 randomly chosen New York workers:

5	10	15	15	15	15	20	20	20	20	30	30	40	40	45	60	60	65	85

2. Find the mean of the New York travel times.

Solution:

$$mean = \frac{5 + 10 + 15 + 15 + 15 + 15 + 20 + 20 + 20 + 20 + 30 + 30 + 40 + 40 + 45 + 60 + 60 + 65 + 85}{19}$$

$$= \frac{615}{19} = 32.1\ minutes$$

3. Compare the two means. Which state has a longer travel time when comparing the means?

Solution:
New York travel times are almost 10 minutes higher than North Carolina travel times.

4. Find the median of the North Carolina and New York travel times.

Median of North Carolina = _____ Median of New York= _____

Solution:
Median of North Carolina = 20 Median of New York= 20

5. Compare the medians. Which state has a longer travel time when comparing the medians?

Solution:
Both states have a median travel time of 20 minutes.

6. Looking at the New York travel times, which number(s) affect the mean, but not the median?

Solution:
The extremely high numbers affect the mean. Numbers such as 65 and 85 pull the mean higher, but do not affect the median. The median is the middle number, and is not influenced by high and low numbers.

7. Create 5 Number Summary for North Carolina and New York.

North Carolina		***New York***	
Minimum	***5***	***Minimum***	***5***
Q1	***10***	***Q1***	***15***
Median	***20***	***Median***	***20***
Q3	***30***	***Q3***	***42.5***
Maximum	***60***	***Maximum***	***85***

Find the Interquartile Range (IQR) of the North Carolina and New York travel times. What does the IQR tell you about each set of data?

North Carolina:
a) Find the distance that upper quartile is away from the lower quartile.

Solution:
Look back at the original values given for North Carolina Drivers.
Find the value of Q3 and Q1. Subtract the value of Q1 from Q3.
30-10= 20
IQR is 20

New York:
a) Find the distance that upper quartile is away from the lower quartile.

Solution:
Look back at the original values of travel times for New York.
Find the value of Q3 and Q1. Subtract the value of Q1 from Q3.
42.5-15= 27.5
IQR is 27.5

8. Overall, which measure of center best describes travel time to work, the mean or the median? Why?

Solution:
The mean is the best measure of center. In this scenario, the mean shows the variation between the travel times better than the median. Since the medians are the same, the median can be deceiving in thinking that the travel times are equivalent for each state.

Travel Times to Work

How long does it take you to get from home to school?

Here are the travel times from home to work in minutes for 15 workers in North Carolina, chosen at random by the Census Bureau:

5	10	10	10	10	12	15	20	20	25	30	30	40	40	60

1. Find the mean of the North Carolina travel times.

Here are the travel times in minutes of 19 randomly chosen New York workers:

5	10	15	15	15	15	20	20	20	20	30	30	40	40	45	60	60	65	85

2. Find the mean of the New York travel times.

3. Compare the two means. Which state has a longer travel time when comparing the means?

4. Find the median of the North Carolina and New York travel times.

   Median of North Carolina = _____ Median of New York= _____

5. Compare the medians. Which state has a longer travel time when comparing the medians?

6. Looking at the New York travel times, which number(s) affect the mean, but not the median?

7. Create 5 Number Summary for North Carolina and New York.

Find the Interquartile Range (IQR) of the North Carolina and New York travel times. What does the IQR tell you about each set of data?

North Carolina:
a) Find the distance that upper quartile is away from the lower quartile.

New York:
a) Find the distance that upper quartile is away from the lower quartile.

8. Overall, which measure of center best describes travel time to work, the mean or the median? Why?

Thumbs on Fire Part 2

Back to task table

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- What are ways to summarize numerical data sets?

MATERIALS REQUIRED

Thumbs on Fire article http://www.cnn.com/2014/05/15/tech/mobile/guiness-record-fastest-text/

Teacher Notes

This task is designed around a specific context (the world record for texting) while allowing teachers the flexibility of developing the procedures. In the task description below, one way of implementing this task is described although there are many different ways in which this context could be used in a Math 7 class.

In this task, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on the board and on student recording sheet. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it and it will be given to them.

Task Description

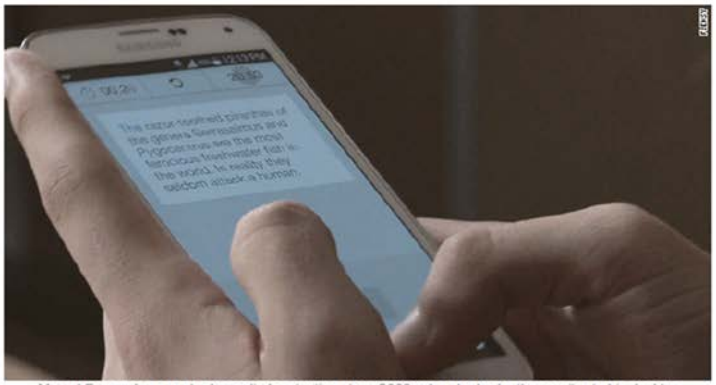
Thumbs on Fire (Part 2)

Have the students read the following article as an introduction to Part 2: Thumbs on Fire taken from http://www.cnn.com/2014/05/15/tech/mobile/guiness-record-fastest-text/

Teen breaks record for fastest text

By Doug Gross , CNN
updated 3:55 PM EDT, Thu May 15, 2014 | Filed under: Mobile

cnn.com



Marcel Fernandes says he has relied on texting since 2009, when he broke the monitor to his desktop computer.

(CNN) -- A 16-year-old from Brazil can dash off a wordy, complicated, text message in the time most of us can thumb "Where R U?" And the folks with Guinness World Records have noticed.

Marcel Fernandes typed a 25-word pre-selected paragraph into his touchscreen phone in 18.19 seconds, just enough to break the record of 18.44 seconds set in January. And that phrase was no easy task, either.

It reads: "The razor-toothed piranhas of the genera Serrasalmus and Pygocentrus are the most ferocious freshwater fish in the world. In reality they seldom attack a human." The entire phrase had to be spelled and punctuated correctly.

Guinness officially acknowledged the record on Thursday. Fernandes set it on April 25 in New York.

Fernandes was flown to New York by Fleksy, makers of the keyboard app he used to break the record. He said he's been using the app since 2012.

"We've been passionate about building the most comfortable, beautiful, and fastest smartphone keyboard ever since we started" says Loannis Verdelis, Fleksy's founder. "The Guinness World Record is a great testament to our progress."

It is clear from the article that companies are working to make smartphone keyboards faster, but which one is fastest and how much faster is one than another? One possible option for this task is for students to investigate two different types of phones and the speed in which a text message can be typed on each phone. For instance, a class may want to compare the length of time it takes a given message to be typed by Apple iPhone users to the length of time it takes Android users to type the same message. (The message in the article is used by Guinness World Records but a different message could be used for this task.)

Some ideas and main questions for this task:

- What 7th grade class is faster at texting?
- Are android or iphone users faster at texting?
- Which middle school grade is the fastest texting grade level
- Have students generate a new 25-word text

Once the main question has been determined, the class will decide on appropriate ways to collect the data.

- Should data be collected only on students in our school or should iPhone and Android users outside our school be included?
- What is an appropriate sample size?

Regardless of the methods used to collect data, the class should engage in a conversation about the validity of the samples taken recalling that conclusions can be drawn about a population only if the sample represents that population.

After the data is collected, students should calculate appropriate measures of center and measures of variability for the samples. Students should compare the two samples and be able to make inferences about the two populations.

This task provides an excellent opportunity for students to utilize mathematical practice 3: *Construct viable arguments and critique the reasoning of others* by writing about their conclusions and providing a critique of their classmates' conclusions.

Formative Assessment Questions

- What models did you create?
- What organizational strategies did you use?
- Does your rule for finding the amount of hair for any weight work? How do you know? Students should read the article *Teen breaks record for fastest text* and discuss possible questions to explore.

Emergency 911! Bay City

Back to task table

*This task requires graphing calculators, but alternate resources can be used.

This task requires that students calculate measures of center and variability to draw conclusions about the effectiveness of two ambulance companies.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

MGSE7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

MGSE7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

ESSENTIAL QUESTION

- How is statistical data used in the real-world?
- How do I use data to make decisions or show evidence of an event occurring or not occurring?

MATERIALS

- Student Task Sheet
- Graphing calculator or TI-30 XII (random number generator is needed)

TASK COMMENTS

Prior to completing the task, the teacher may want to do a mini-lesson with the graphing calculator or TI-30 XII to teach students how to use the random integer function. Options to replace using a calculator include using the computer (random integer site: http://www.random.org/integers/) or put numbers in a container and draw numbers, replacing the number after each draw.

Used with permission from Balanced Assessment: Middle Grades Package 1, Dale Seymour Publications

TASK DESCRIPTION

Last week there was an accident at the Waterfront Amusement Park in Bay City. A seat on one of the rides broke loose, resulting in the deaths of two teenagers. The owners of the amusement park have charged that if ambulances had responded more quickly, the two teens would have survived. They have threatened to sue the Bay City 911 emergency service for failing to dispatch ambulances efficiently.

The Bay City Council has hired your firm to conduct an independent investigation of the city's 911-response to emergencies. The 911 operators dispatch from two companies: Arrow Ambulance and Metro Ambulances.

The 911 operators aren't always sure which company to send when an emergency call is received.

Data on the response times for the two companies for an area of a one-mile radius of the Amusement Park show that responses can take as little as 6 minutes or as long as 19 minutes.

You need to continue your investigation by analyzing the response time data on the next page. Based on the information above and your analysis of the data, you conclude that the Bay City Council needs to establish a policy about which service to call.

Using the steps below, you will discover which Ambulance Service would best serve the Amusement Park.

Incident #	Date of Call	Time of Call	Company Name	Response Time
1.	May 1st	2:20am	Arrow	11
2.	May 1st	12:41pm	Arrow	8
3.	May 3rd	6:23pm	Arrow	19
4.	May 4th	7:12am	Arrow	7
5.	May 6th	12:22pm	Arrow	16
6.	May 6th	6:47am	Arrow	8
7.	May 9th	5:37pm	Arrow	17
8.	May 9th	9:37pm	Arrow	6
9.	May 10th	6:25am	Arrow	16
10.	May 13th	6:40am	Arrow	17
11.	May 17th	11:09am	Arrow	7
12.	May 17th	9:15pm	Arrow	8
13.	May 20th	7:25am	Arrow	17
14.	May 23rd	2:39pm	Arrow	10
15.	May 25th	8:30pm	Arrow	8
16.	May 27th	4:21pm	Arrow	9

Incident #	Date of Call	Time of Call	Company Name	Response Time
1.	May 1st	8:14am	Metro	11
2.	May 2nd	6:23am	Metro	8
3.	May 2nd	4:15am	Metro	16
4.	May 3rd	8:41am	Metro	7
5.	May 4th	4:15am	Metro	11
6.	May 4th	8:41am	Metro	11
7.	May 5th	7:43am	Metro	12
8.	May 5th	10:02pm	Metro	9
9.	May 7th	7:15am	Metro	16
10.	May 7th	6:10pm	Metro	11
11.	May 11th	1:03am	Metro	12
12.	May 13th	3:25pm	Metro	15
13.	May 14th	4:59pm	Metro	14
14.	May 16th	10:11am	Metro	8
15.	May 16th	11:45am	Metro	10
16.	May 17th	11:15pm	Metro	8
17.	May 20th	4:20pm	Metro	19
18.	May 23rd	3:44pm	Metro	14
19.	May 24th	8:56pm	Metro	10
20.	May 26th	6:33am	Metro	6

Randomly select five response times from Arrow Ambulances.

- To find five random responses, you can use the Random Integer function on the calculator (TI-30X II: press PRB, then RANDI (1, 16), press enter, one incident number will appear) or put the numbers (1-16) in a container, draw a number, put number back in container and repeat.
- Look up the corresponding incident number and record the response time below.
- Continue until you have found five response times for Arrow Ambulances.

***Teacher Commentary*: If you do not have access to enough calculators, you have options. You can use 1 calculator and choose samples for everyone to use in class. This may make grading the task easier if all students use the same values. Or, you can also put numbers in a bucket and draw values from the bucket. Replace numbers once you have a group completed.**

Response times for 5 random Arrow incidents:
Randomized Group 1

Random integers					
Corresponding Time					

Sample Solution:

Random integers	*10*	*1*	*5*	*11*	*7*
Corresponding Time minutes	*17*	*11*	*16*	*7*	*17*

1. Find 5 more random incidents and response times for Arrow Ambulances. Use the procedure above, or continue to press enter to get 5 more random incidents for the groups below. It is okay if an incident number repeats in one group.

Response times for 5 random Arrow incidents:
Randomized Group 2

Random integers					
Corresponding Time					

Sample Solution:

Random integers	*10*	*8*	*1*	*6*	*13*
Corresponding Time minutes	*17*	*6*	*11*	*8*	*17*

Response times for 5 random Arrow incidents:
Randomized Group 3

Random integers					
Corresponding Time					

Sample Solution:

Random integers	*4*	*3*	*16*	*7*	*8*
Corresponding Time minutes	*7*	*19*	*9*	*17*	*6*

Response times for 5 random Arrow incidents:
Randomized Group 4

Random integers					
Corresponding Time					

Sample Solution:

Random integers	*15*	*6*	*4*	*1*	*5*
Corresponding Time minutes	*8*	*8*	*7*	*11*	*16*

Response times for 5 random Arrow incidents:
Randomized Group 5

Random integers					
Corresponding Time					

Sample Solution:

Random integers	*12*	*16*	*8*	*9*	*15*
Corresponding Time minutes	*8*	*9*	*6*	*16*	*8*

2. Find the mean response time for Groups 1-5 above.
   Then find the mean of the five groups mean response times.

Group	Mean
1	
2	
3	
4	
5	
Total Mean of Groups 1-5 "the mean of the means"	

Sample Solution:

Group	Mean
1	*68/5=13.6*
2	*59/5=11.8*
3	*58/5=11.6*
4	*50/5=10*
5	*47/5=9.4*
Total Mean of Groups 1-5 "the mean of the means"	$\frac{13.6+11.8+11.6+10+9.4}{5}=\frac{56.4}{5}=11.28\ minutes$

3. Find the mean response time of all 16 Arrow Ambulances. Compare that to the "mean of the means" from question 3. Are they similar or different? Why do you think that is?

Solution: ***The mean of all 16 Arrow incidents is 11.5 minutes. The mean of all 16 ambulance incidents is very similar to the "mean of the means" of 11.28. Since they use the same values and they are both means, they will be similar.***

Now, you will complete the same procedure with Metro Ambulances.

4. Randomly select five response times from Metro Ambulances.
   - To find five random responses, you can use the Random Integer function on the calculator (TI-30X II: press PRB, then RANDI (1, 16), press enter, one incident number will appear) or put the numbers (1-16) in a container, draw a number, put number back in container and repeat.
   - Look up the corresponding incident number and record the response time below.
   - Continue until you have found five response times for Metro Ambulances.

***Teacher Commentary**: If you do not have access to enough calculators, you have options. You can use 1 calculator and choose samples for everyone to use in class. This may make grading the task easier if all students use the same values. Or, you can also put numbers in a bucket and draw values from the bucket. Replace numbers once you have a group completed.*

Response times for 5 random Metro incidents:
Randomized **Group 1**

Random integers					
Corresponding Time					

Sample Solution:

Random integers	*9*	*2*	*6*	*12*	*10*
Corresponding Time minutes	*16*	*8*	*11*	*15*	*11*

5. Find 5 more random incidents and response times for Metro Ambulances. Use the procedure above, or continue to press enter to get 5 more random incidents for the groups below. It is okay if an incident number repeats in one group.

Response times for 5 random Metro incidents:
Randomized **Group 2**

Random integers					
Corresponding Time					

Sample Solution:

Random integers	*14*	*16*	*1*	*11*	*2*
Corresponding Time minutes	*8*	*8*	*11*	*12*	*8*

Response times for 5 random Metro incidents:
Randomized **Group 3**

Random integers					
Corresponding Time					

Sample Solution:

Random integers	*6*	*13*	*12*	*5*	*3*
Corresponding Time minutes	*11*	*14*	*15*	*11*	*16*

Response times for 5 random Metro incidents:
Randomized **Group 4**

Random integers					
Corresponding Time					

Sample Solution:

Random integers	*2*	*19*	*11*	*6*	*17*
Corresponding Time minutes	*8*	*10*	*12*	*11*	*19*

Response times for 5 random Metro incidents:
Randomized **Group 5**

Random integers					
Corresponding Time					

Sample Solution:

Random integers	*3*	*19*	*15*	*5*	*7*
Corresponding Time minutes	*16*	*10*	*10*	*11*	*12*

6. Find the mean response time for Groups 1-5 above.
   Then find the mean of the five groups mean response times.

Group	Mean
1	
2	
3	
4	
5	
Total Mean of Groups 1-5 "the mean of the means"	

Sample Solution:

Group	Mean
1	*61/5=12.2*
2	*47/5=9.4*
3	*67/5=13.4*
4	*60/5=12*
5	*59/5=11.8*
Total Mean of Groups 1-5 "the mean of the means"	$\frac{12.2+9.4+13.4+12+11.8}{5}=\frac{58.8}{5}=11.76$

7. Find the mean response time of all 20 Metro Ambulances. Compare that to the "mean of the means" from question 7. Are they similar or different? Why do you think that is?

Solution:
Mean of all 20 ambulance incidents is 10.9 minutes. Compared to the "mean of the means" in question 7, the mean of all incidents is almost a full minute different. 1 minute could mean a huge difference in emergency situations, so 10.9 and 11.76 are quite different.

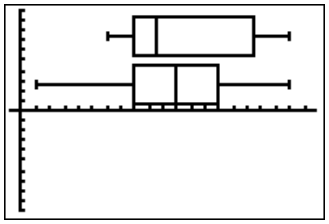
8. Did either ambulance company have outliers, numbers that stood out to be extremely high or extremely low? How do outliers affect the mean response time?

Solution:
No outliers were apparent in any incident from Arrow or Metro. Outliers can pull the mean response time higher or lower. No outliers appeared to affect the means for Arrow or Metro.

Comments:
Some students may want for "19" to be an outlier in Arrow or Metro. But since both ambulance companies have other incidents within 2-3 minutes of 19, it is not a "strong" outlier.

9. The Bay City Council wants your recommendation of which company to use. Prepare a statistical chart below (dot plot, histogram, box plot, bar graph, etc) to support your recommendation. Then be sure to give clear, statistical reasons for the ambulance company you are recommending.

Solution:
Metro Ambulances should be chosen based on the lower mean of 10.9 minutes as compared to Arrow's 11.9 minute mean.
The box plots show Arrow at the top, and Metro under it. The overall "box" of the box plots shows Metro as having a trend for lower response times.



Since minutes and seconds are crucial during emergency situations, the ambulance service with the overall lower average response time should be chosen by the Amusement Park as their preferred ambulance carrier.

Emergency 911! Bay City!

Last week there was an accident at the Waterfront Amusement Park in Bay City. A seat on one of the rides broke loose, resulting in the deaths of two teenagers. The owners of the amusement park have charged that if ambulances had responded more quickly, the two teens would have survived. They have threatened to sue the Bay City 911 emergency service for failing to dispatch ambulances efficiently.

The Bay City Council has hired your firm to conduct an independent investigation of the city's 911 response. The 911 operators dispatch from two companies: Arrow Ambulance and Metro Ambulances.

The 911 operators aren't always sure which company to send when an emergency call is received.

Data on the response times for the two companies for an area of a one-mile radius of the Amusement Park show that responses can take as little as 6 minutes or as long as 19 minutes.

You need to continue your investigation by analyzing the response time data on the next page. Based on the information above and your analysis of the data, you conclude that the Bay City Council needs to establish a policy about which service to call.

Using the steps below, you will discover which Ambulance Service would best serve the Amusement Park.

Incident #	Date of Call	Time of Call	Company Name	Response Time
1.	May 1st	2:20am	Arrow	11
2.	May 1st	12:41pm	Arrow	8
3.	May 3rd	6:23pm	Arrow	19
4.	May 4th	7:12am	Arrow	7
5.	May 6th	12:22pm	Arrow	16
6.	May 6th	6:47am	Arrow	8
7.	May 9th	5:37pm	Arrow	17
8.	May 9th	9:37pm	Arrow	6
9.	May 10th	6:25am	Arrow	16
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11.	May 17th	11:09am	Arrow	7
12.	May 17th	9:15pm	Arrow	8
13.	May 20th	7:25am	Arrow	17
14.	May 23rd	2:39pm	Arrow	10
15.	May 25th	8:30pm	Arrow	8
16.	May 27th	4:21pm	Arrow	9

Incident #	Date of Call	Time of Call	Company Name	Response Time
1.	May 1st	8:14am	Metro	11
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4.	May 3rd	8:41am	Metro	7
5.	May 4th	4:15am	Metro	11
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13.	May 14th	4:59pm	Metro	14
14.	May 16th	10:11am	Metro	8
15.	May 16th	11:45am	Metro	10
16.	May 17th	11:15pm	Metro	8
17.	May 20th	4:20pm	Metro	19
18.	May 23rd	3:44pm	Metro	14
19.	May 24th	8:56pm	Metro	10
20.	May 26th	6:33am	Metro	6

1. Randomly select five response times from Arrow Ambulances.
   - To find five random responses, you can use the Random Integer function on the calculator (TI-30X II: press PRB, then RANDI (1, 16), press enter, one incident number will appear) or put the numbers (1-16) in a container, draw a number, put number back in container and repeat.
   - Look up the corresponding incident number and record the response time below.
   - Continue until you have found five response times for Arrow Ambulances.

Response times for 5 random Arrow incidents:
Randomized **Group 1**

Random integers					
Corresponding Time					

2. Find 5 more random incidents and response time for Arrow Ambulances. Use the procedure above, or continue to press enter to get 5 more random incidents for the groups below. It is okay if an incident number repeats in one group.

Response times for 5 random Arrow incidents:

Randomized **Group 2**

Random integers					
Corresponding Time					

Response times for 5 random Arrow incidents:
Randomized **Group 3**

Random integers					
Corresponding Time					

Response times for 5 random Arrow incidents:
Randomized **Group 4**

Random integers					
Corresponding Time					

Response times for 5 random Arrow incidents:
Randomized **Group 5**

Random integers					
Corresponding Time					

3. Find the mean response time for Groups 1-5 above.
   Then find the mean of the five groups mean response times.

Group	Mean
1	
2	
3	
4	
5	
Total Mean of Groups 1-5 "the mean of the means"	

4. Find the mean response time of <u>all 16 Arrow Ambulances</u>. Compare that to the "mean of the means" from question 3. Are they similar or different? Why do you think that is?

Now, you will complete the same procedure with Metro Ambulances.

5. Randomly select five response times from Metro Ambulances.
   - To find five random responses, you can use the Random Integer function on the calculator (TI-30X II: press PRB, then RANDI (1, 16), press enter, one incident number will appear) or put the numbers (1-16) in a container, draw a number, put number back in container and repeat.
   - Look up the corresponding incident number and record the response time below.
   - Continue until you have found five response times for Metro Ambulances.

Response times for 5 random Metro incidents:
Randomized **Group 1**

Random integers					
Corresponding Time					

6. Find 5 more random incidents and response time for Metro Ambulances. Use the procedure above, or continue to press enter to get 5 more random incidents for the groups below. It is okay if an incident number repeats in one group.

Response times for 5 random Metro incidents:
Randomized **Group 2**

Random integers					
Corresponding Time					

Response times for 5 random Metro incidents:
Randomized **Group 3**

Random integers					
Corresponding Time					

Response times for 5 random Metro incidents:
Randomized **Group 4**

Random integers					
Corresponding Time					

Response times for 5 random Metro incidents:
Randomized **Group 5**

Random integers					
Corresponding Time					

7. Find the mean response time for Groups 1-5 above.
   Then find the mean of the five groups mean response times.

Group	Mean
1	
2	
3	
4	
5	
Total Mean of Groups 1-5 "the mean of the means"	

8. Find the mean response time of <u>all 20 Metro Ambulances</u>. Compare that to the "mean of the means" from question 7. Are they similar or different? Why do you think that is?

9. Did either ambulance company have outliers, numbers that stood out to be extremely high or extremely low? How do outliers affect the mean response time?

10. The Bay City Council wants your recommendation of which company to use. Prepare a statistical chart below (dot plot, histogram, box plot, bar graph, etc) to support your recommendation. Then be sure to give clear, statistical reasons for the ambulance company you are recommending.

TECHNOLOGY RESOURCES

Back to task table

Use random sampling to draw inferences about a population.

MGSE7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

http://nzmaths.co.nz/resource/planning-statistics-investigation-l3

http://nzmaths.co.nz/resource/channel-surfing

http://nzmaths.co.nz/resource/stork-delivery

http://illuminations.nctm.org/Activity.aspx?id=3580

MGSE7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

http://nzmaths.co.nz/resource/planning-statistics-investigation-l3
http://nzmaths.co.nz/resource/channel-surfing
http://nzmaths.co.nz/resource/stork-delivery
http://illuminations.nctm.org/Activity.aspx?id=3580

Draw informal comparative inferences about two populations.

MGSE7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the medians by expressing it as a multiple of the interquartile range.

https://www.illustrativemathematics.org/content-standards/7/SP/B/3/tasks

https://www.illustrativemathematics.org/content-standards/7/SP/B/4/tasks

http://nzmaths.co.nz/resource/number-statistics

http://nzmaths.co.nz/resource/sports-statistics

http://nzmaths.co.nz/illustrating-mathematics-standards-60

http://illuminations.nctm.org/Activity.aspx?id=3576

http://illuminations.nctm.org/Activity.aspx?id=3580

http://illuminations.nctm.org/Lesson.aspx?id=2643

http://illuminations.nctm.org/Activity.aspx?id=3580

MGSE7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

https://www.illustrativemathematics.org/content-standards/7/SP/B/3/tasks

https://www.illustrativemathematics.org/content-standards/7/SP/B/4/tasks

http://nzmaths.co.nz/resource/number-statistics

http://nzmaths.co.nz/resource/sports-statistics

http://nzmaths.co.nz/illustrating-mathematics-standards-60

http://illuminations.nctm.org/Activity.aspx?id=3576

http://illuminations.nctm.org/Lesson.aspx?id=2643

http://illuminations.nctm.org/Activity.aspx?id=3580