



Rate of Change and Slope

Essential question: *How do you find a rate of change or a slope?*

A rate of change is a ratio of the amount of change in the output to the amount of change in the input.

COMMON CORE

CC.8.EE.5

1 EXAMPLE Investigating Rates of Change

Eve keeps a record of the number of lawns she mows and the money she earns.

	Day 1	Day 2	Day 3	Day 4	Day 5
Number of Lawns	1	3	6	8	13
Amount Earned (\$)	15	45	90	120	195

Input variable: _____ Output variable: _____

Find the rates of change:

$$\text{Day 1 to Day 2} \quad \frac{\text{change in \$}}{\text{change in lawns}} = \frac{45 - 15}{3 - 1} = \frac{30}{2} = 15$$

$$\text{Day 2 to Day 3} \quad \frac{\text{change in \$}}{\text{change in lawns}} = \frac{90 - 45}{6 - 3} = \frac{45}{3} = 15$$

$$\text{Day 3 to Day 4} \quad \frac{\text{change in \$}}{\text{change in lawns}} = \frac{120 - 90}{8 - 6} = \frac{30}{2} = 15$$

$$\text{Day 4 to Day 5} \quad \frac{\text{change in \$}}{\text{change in lawns}} = \frac{195 - 120}{13 - 8} = \frac{75}{5} = 15$$

The rates of change are constant / variable.

TRY THIS!

1. The table shows the approximate height of a football after it is kicked.

Time (s)	0	0.5	1.5	2
Height (ft)	0	18	31	26

Input variable: _____ Output variable: _____

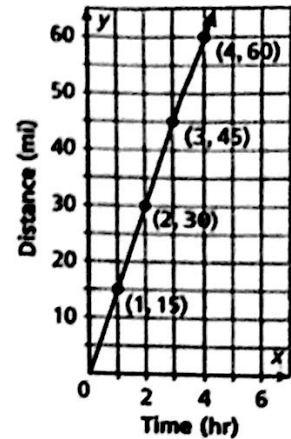
Find the rates of change:

The rates of change are constant / variable.

You can also use a graph to find rates of change.

2 EXPLORE Using Graphs to Find Rates of Change

The graph shows the distance Nathan bicycled over time. What is Nathan's rate of change?



- A Find the rate of change from 1 hour to 2 hours.

$$\frac{\text{change in distance}}{\text{change in hours}} = \frac{30 - 15}{2 - 1} = \frac{15}{1} = 15 \text{ miles per hour}$$

- B Find the rate of change from 1 hour to 4 hours.

$$\frac{\text{change in distance}}{\text{change in hours}} = \frac{60 - 15}{4 - 1} = \frac{45}{3} = 15 \text{ miles per hour}$$

- C Recall that the graph of a proportional relationship is a straight line through the origin. Explain whether the relationship between Nathan's time and distance appears to be a proportional relationship.

- D Find Nathan's unit rate.

- E Compare the rate of change to the unit rate.

REFLECT

- 2a. Does it matter what interval you use when you find the rate of change of a proportional relationship? Explain.

- 2b. **Conjecture** Do you think that the value of r in the point $(1, r)$ is always the unit rate for any situation? Explain.

1. Gerri dropped a ball from the top of a building. Use the table to find the rate of change over each interval.

Time (s)	0	1	2	3	4
Height of Ball (ft)	256	240	192	112	0

a. 0 seconds to 1 second _____

b. 3 seconds to 4 seconds _____

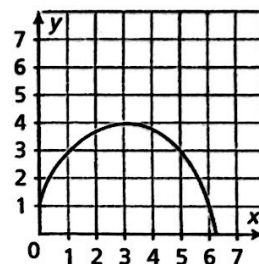
c. 1 second to 3 seconds _____

2. Is the ball's rate of change constant or variable?

Use the graph to find the average rate of change over each interval.

3. $x = 0$ to $x = 3$

4. $x = 5$ to $x = 6$

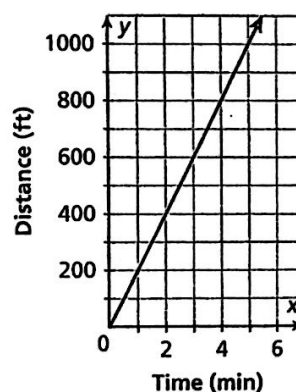


Erica walks to her friend Philip's house at a constant pace. The graph shows Erica's distance from home over time.

5. Without calculating slope, tell whether the slope is positive or negative.

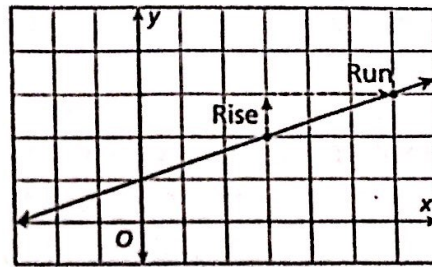
6. Find the slope of the line.

7. Does the value of r in the point $(1, r)$ correspond to Erica's unit rate? Explain.



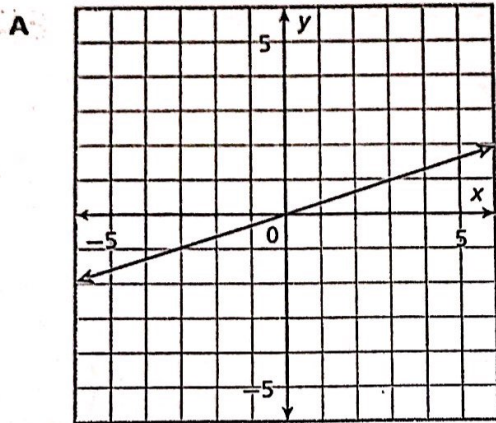
When the rate of change of a relationship or function is constant, every segment of its graph has the same steepness and together they form a straight line. The constant rate of change is called the *slope* of the line.

The slope of a line is the ratio of the change in *y*-values (rise) for a segment of the graph to the corresponding change in *x*-values (run).



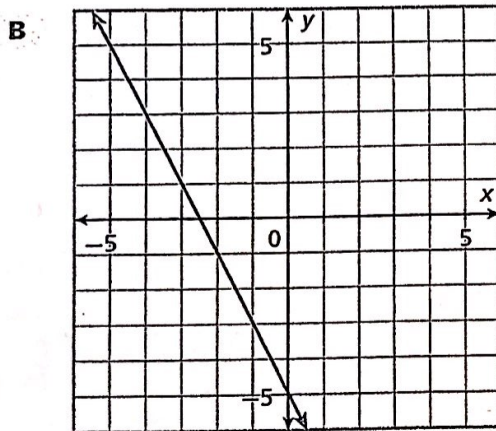
3 EXAMPLE Calculating Slope

Find the slope of the line.



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{2}{10} = \frac{1}{5}$$



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{-10}{10} = -1$$

REFLECT

3a. If a line rises from left to right, what is the sign of its slope? If a line falls from left to right, what is the sign of its slope?

3b. What type of line has a slope equal to 0?

3c. What happens when you try to calculate the slope of a vertical line?
